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NONUNIFORMLY OFFSET POLYPHASE SYNTHESIS OF A BANDPASS SIGNAL FROM COMPLEX-ENVELOPE SAMPLES

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ABSTRACT

In this paper we consider the synthesis of a bandpass signal from complex-envelope samples using a polyphase conversion structure based on periodically nonuniform output samples. This approach provides the flexibility to independently choose the sampling rate and carrier frequency, overcoming the restrictions of reconstruction from uniformly spaced output samples. Key to the design of such a system is a particular equivalent filter with a piecewise-constant impulse response that determines both the actual implementation filters and system performance. The transition times of this impulse response are found to be periodically nonuniform, leading to a characterization of the corresponding frequency response. An example design using a previously unavailable carrier frequency is presented which requires fewer filter taps than the alternative, a higher-rate uniform system.

1. INTRODUCTION

When an analog bandpass signal is synthesized from complex-envelope samples through D/A conversion at a uniform sample rate (uniform synthesis), carrier-frequency choices are limited. Here this limitation is overcome through the nonuniformly timed interleaving of polyphase data streams prior to D/A conversion (nonuniform synthesis). After describing the system, we consider the design of the equivalent filter that is key to understanding its operation.

In the synthesis/modulation system of Fig. 1(a), the real parts of the outputs of M discrete-time filters are nonuniformly interleaved for D/A conversion and bandpass filtering. Except for the output timing, these M filtering arms amount to the polyphase components of an interpolation filter. The D/A receives samples from filter k at times $\mathbb{Z}/f_s + \alpha_k$ and outputs a (typically) rectangular pulse of width T_k. The result is a stairstep waveform with periodically nonuniform step widths (and possibly gaps between steps). In the mathematically equivalent system of Figure 1(b), sequence values are (conceptually) mapped to impulses and passed through a single equivalent filter whose complex piecewise-constant impulse response is related to that of the component filters as illustrated in Figure 2. Figure 3 shows representative signals. After the desired spectral component is shifted to the carrier frequency, the filters must together suppress the other components (with optional spectral shaping). The bandpass signal is then just the real part.

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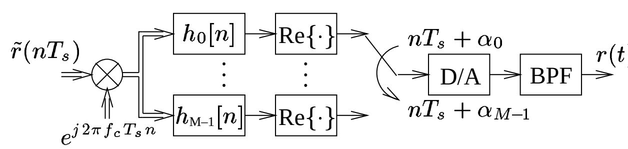
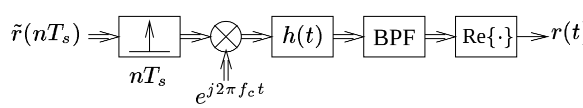


(a) Modulation using periodically nonuniform synthesis



(b) Modulation using uniform synthesis and a piecewise-constant filter with nonuniformly spaced transitions

Figure 1: Modulator system (a) is for implementation. Equivalent modulator system (b) is for analysis and design.

Systems which perform uniform bandpass signal synthesis directly from complex-envelope samples are becoming more commonplace in the literature [1–4]. Nonuniform bandpass synthesis is briefly mentioned in [5], as it is closely related to the well-studied problem of nonuniform bandpass sampling [6, 7].

2. DESIGNING THE EQUIVALENT FILTER

The analysis in this section does not provide a design method for the equivalent filter of Fig. 1(b), but rather a characterization of the available spectral responses of such a piecewise-constant filter. This then guides the selection of the offsets α_k so that FIR polyphase component filters of modest lengths will suffice. If the polyphase component filters are FIR, the frequency response of the equivalent filter will turn out to be, at any particular frequency, linear in the filter coefficients, so the filters can be designed with linear programming, second-order cone programming (SOCP) [8], or generalized-eigenfilter methods, for example.

The frequency response of a filter with a piecewise-constant impulse response is closely related to the response of a nonuniform tapped-delay-line (TDL) filter. Such a TDL response arises in nonuniform bandpass sampling [5]. The key difference is the addition of a hold response in each polyphase arm, which results in an overall response that replaces the impulses of each polyphase TDL impulse response with copies of the corresponding hold response. Defining $\tau_k \equiv \alpha_k + T_k/2$, the centers of the impulse response stairsteps have support in $\mathcal{T} + \mathbb{Z}/f_s$, where $\mathcal{T} = \{\tau_0, \ldots, \tau_{M-1}\}$

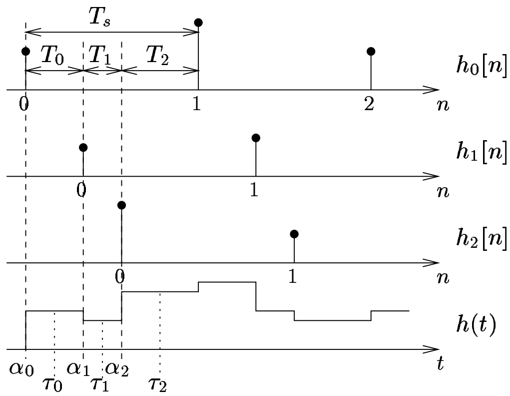


Figure 2: The synthesis equivalent filter is related to (discrete-time representations of) its polyphase component filters as shown.

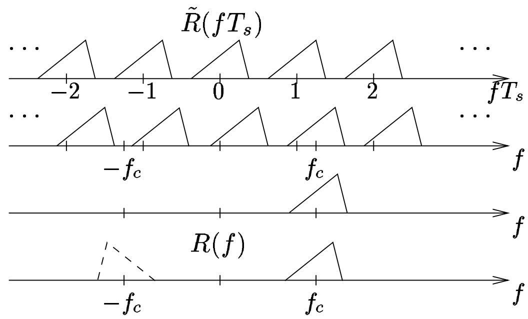


Figure 3: Example signal spectra for the system of Fig. 1(b).

(see Fig. 2). The frequency response of such a filter can be written as

$$H(f) = \sum_{k=0}^{M-1} H_k(f) P_k(f) e^{-j2\pi f \tau_k}, \tag{1}$$

where $H_k(f)$ is the period f_s response of a uniform TDL filter, and $P_k(f)$ is the (zero-phase) response of a time-symmetric zero-order hold of duration T_k.

A useful concept when dealing with periodic functions such as uniform TDL frequency responses is the $\mathrm{aliasfree}(f_s)$ zone, defined in [7] as the spectral support of a signal that can be sampled at a rate f_s without aliasing. From a filter-design viewpoint, an $\mathrm{aliasfree}(f_s)$ zone represents a set of frequencies on which a periodic frequency response with period f_s can be specified without conflict. Since $\mathcal{F}$ is $\mathrm{aliasfree}(f_s)$ if no two frequencies in $\mathcal{F}$ are equal modulo f_s, it follows that

$$\mathcal{F}_m \equiv \{ f + m(f) f_s : f \in \mathcal{F} \} \tag{2}$$

is also $\mathrm{aliasfree}(f_s)$ for any function $m : \mathcal{F} \to \mathbb{Z}$. Specifying a periodic response on $\mathcal{F}_m$ is equivalent to specifying it on $\mathcal{F}$.

For integer-valued function $m(f)$ then,

$$H(f + m(f)f_s) = \sum_{k=0}^{M-1} H_k(f) P_k(f + m(f)f_s) \gamma_k^{m(f)} e^{-j2\pi f \tau_k},$$

where $\gamma_k = e^{-j2\pi f_s \tau_k}$. This can be written simultaneously for $m(f) = m_0(f), \ldots, m_{M-1}(f)$ as the matrix equation

$$\begin{pmatrix} H(f + m_0(f)f_s) \\ \vdots \\ H(f + m_{M-1}(f)f_s) \end{pmatrix} = \Gamma(f) D(f) \begin{pmatrix} H_0(f) \\ \vdots \\ H_{M-1}(f) \end{pmatrix}, \tag{3}$$

with matrices $\Gamma(f)$, $D(f)$ defined by

$$[\Gamma]_{i,j} = \gamma_j^{m_i(f)} P_j(f + m_i(f) f_s) \tag{4}$$

for $i, j = 0, \ldots, M-1$, and

$$D(f) = \mathrm{diag}(e^{-j2\pi f \tau_0}, \ldots, e^{-j2\pi f \tau_{M-1}}).$$

If f is restricted to some $\mathrm{aliasfree}(f_0)$ zone $\mathcal{F}$, say $(-f_s/2, f_s/2)$, then (3) relates the equivalent response $H(f)$ on M $\mathrm{aliasfree}(f_s)$ regions $\mathcal{F}_{m_0}, \ldots, \mathcal{F}_{m_{M-1}}$ determined by (2) to the frequency responses on $\mathcal{F}$ of the polyphase component filters $H_0, \ldots, H_{M-1}$ through matrices $\Gamma(f)$ and $D(f)$.

Matrix $D(f)$ is unitary and hence invertible, so if $\Gamma(f)$ is also invertible for every $f \in \mathcal{F}$, then the M responses of the polyphase component filters can be determined from the M segment responses of filter $H(f)$. If Γ is singular at some frequency f its range space is of dimension less than M, and some of those equivalent-filter segment responses can be determined from the others at that frequency; independent specification of $H(f)$ on this family of segments is not possible. Further, for any f the ratio of the L_2 norm of the vector of segment responses to the L_2 norm of the vector of polyphase-component responses is bounded between the maximum and minimum singular values of $\Gamma(f)$. If these values differ widely—condition number $\chi(\Gamma)$, the ratio of the maximum to minimum singular value, provides a measure—extreme behavior may be required of the polyphase component filters in order to effect modest equivalent-filter behavior. Because of the hold responses inherent in D/A conversion, the condition number of $\Gamma(f)$ will be a function of frequency even when $m_0(f), \ldots, m_{M-1}(f)$ are constant. This stands in contrast to the otherwise similar nonuniform sampling [5], where the special case of uniform sampling (for example) results in all singular values being identical.

A short summary of general design principles can now be stated. Choose arm-number M to provide the necessary total design bandwidth, and choose functions $m_k(f)$ (relative to some $\mathcal{F}$) to define the desired spectral regions to be designed. For bandpass synthesis the region to be designed is usually two bandpass intervals centered at the positive and negative carrier frequencies. For even M it is always possible to so choose $m_k(f)$ for any carrier frequency, simply by designating half of the functions $m_k(f)$ for the positive interval and half for the negative. For odd M this is only possible when $f_c \in \frac{2\mathbb{N}-1}{4} f_s$, which is a consequence of splitting one of the integer functions between bands. Other choices are possible when the carrier is not required to be the center of the design interval. Choosing the timing parameters τ_k and T_k generally involves a tradeoff between performance and system complexity — the optimal set of timings (those that minimize the condition number of (4)) may be unrealistic and would usually be approximated by a ratio of small integers. The requirement that the output of one arm not overlap another (thus allowing implementation with a single D/A) implies that T_k is not independent of the choice of τ_k and in general results in gaps in the impulse response and the D/A output. Since closely spaced τ_k imply small T_k, nonuniform synthesis may not lessen the conversion-time requirements on the D/A as compared to higher output-rate uniform synthesis.

Having determined suitable timing parameters, one could solve (3) for "ideal" arm filters and then seek to approximate those responses in some way. A much more powerful approach is to use (1) directly, implicitly designing the arm-filter responses while optimizing some measure of system performance. An example will illustrate this approach.

3. DESIGN EXAMPLE

Consider a bandpass synthesis system operating at an input sample rate of f_s on a complex envelope of two-sided bandwidth $0.8f_s$, with a carrier frequency of $1.25f_s$. The last is an interesting choice for several reasons. First, it will allow full design control with an odd value of M. Second, for $M = 3$ this carrier frequency is unavailable to a uniform system (due to the filter periodicity). Lastly, this carrier frequency represents the classic choice of $f_c = Mf_s/4$ for a uniform system with $M = 5$, allowing a comparison of a nonuniform system with a higher-rate uniform system. Figure 4 illustrates a possible choice for the functions m_0, m_1, m_2 and the resulting spectral regions $\mathcal{F}_{m_0}, \mathcal{F}_{m_1}, \mathcal{F}_{m_2}$ when $M = 3$.

Key to the operation of a bandpass synthesis system is the analog bandpass filtering at the output, which is responsible for removing the (often significant) out-of-band signals from the D/A. From the equivalent-filter viewpoint the bandpass filter suppresses frequencies outside of the design regions $\mathcal{F}_{m_0}, \ldots, \mathcal{F}_{m_{M-1}}$. These systems often require relatively high-order filters due to the typically high ratio of bandwidth to carrier frequency. The examples used a Chebychev type-I filter created from a 6th-order lowpass prototype with a bandwidth of f_s and ± 0.5 dB of passband ripple. The passband edges, located at $(0.7f_s, 1.7f_s)$, were chosen experimentally to lessen group-delay effects in the design band while keeping the bandwidth modest. The equivalent filter must have nonlinear phase to correct the remaining phase errors.

Now comes the matter of choosing the timing parameters. Using (4) and (for simplicity) temporarily neglecting the hold functions, the minimum condition number of Γ was found to be 1.815, corresponding to $\mathcal{T} = \{-0.217, 0, 0.217\}$. This set is not attainable with maximum-duration hold responses, and so two choices of parameters were tried. For case 1, transition times $\{\alpha_k\} = \{-0.5, -0.25, 0.25\}$ were used with full-duration holds $\{T_k\} = \{0.25, 0.5, 0.25\}$, yielding $\mathcal{T} = \{-0.375, 0, 0.375\}$, which is a poor approximation and shows the drawback of using full hold durations. Case 2 uses the better choice $\mathcal{T} = \{-0.2, 0, 0.2\}$ with $\{T_k\} = \{0.2, 0.2, 0.2\}$, which represents a subset of uniform timings with $M = 5$. Both of these will be compared to case 3, a full uniform $M = 5$ system. Fig. 5 shows the condition number of (4) vs. frequency for case 1 and case 2. As expected it is small and nearly constant for case 2, but singular at $f = 0$ for case 1, resulting in an infinite condition number at that point. This corresponds to the frequencies $\{-f_s, f_s, 2f_s\}$ in $\mathcal{F}_{m_0}, \mathcal{F}_{m_1}, \mathcal{F}_{m_2}$. In general attempts to independently fix the response at all 3 frequencies will fail, since any two determine the third.

Conceptually, the filter design process for the three example systems was straightforward, using a SOCP engine to perform constrained optimization of the cascade of the analog filter and (1). A 40 dB L_∞ stopband constraint was placed at frequencies below $0.65f_s$ and above $1.85f_s$, ensuring that all spectral replicas of the input signal are suppressed at least 40 dB. Mean-square error (with respect to unity) in the passband region $[0.85f_s, 1.65f_s]$ was then

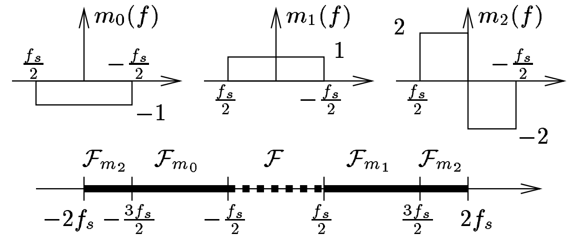


Figure 4: The functions m_0, m_1, m_2 defined on $\mathcal{F} = [-\frac{f_s}{2}, \frac{f_s}{2}]$, and the corresponding spectral regions $\mathcal{F}_{m_0}, \mathcal{F}_{m_1}, \mathcal{F}_{m_2}$.

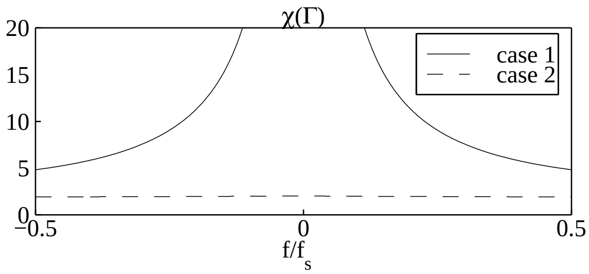


Figure 5: Condition number $\chi(\Gamma(f))$ versus frequency for the two nonuniform cases.

minimized. Auxiliary constraints were used as needed to prevent high FIR filter gain in transition regions. Fig. 6 shows the resulting responses for a total of 24 complex coefficients per system.

Comparing the two nonuniform systems shows that, as predicted by the condition numbers of the Γ matrices, much more extreme behavior is seen in case 1 than case 2, in both the individual filters and the equivalent-filter response. Whereas the maximum equivalent-filter out-of-band gain for case 2 is only slightly above the passband, for case 1 it is 20 dB higher and cannot be brought down without further distorting the passband. This is unlikely to be acceptable in a practical system, as it reduces the available dynamic range (in the D/A) for passband signals. Further, examining the passband shows a large ripple at $f = f_s$, the result of the aforementioned conflicting stopband constraints at $f = \{-f_s, 2f_s\}$. Improving passband performance here would require relaxing the stopband constraint at one of the other frequencies.

Comparing cases 2 and 3 shows the potential savings of the nonuniform architecture. Both cases produce similar cascade responses, but the nonuniform response has only about $1/4$th as much error. Equivalently, as shown in Fig. 7 for a range of filter lengths, the nonuniform system requires about $1/3$ fewer taps for the same performance as the uniform system.

4. CONCLUSIONS

The performance benefits of the nonuniform system can be intuitively understood from the impulse response plots of Fig. 6. Since the tap density of the equivalent filter is lower for (nonuniform) case 2 than for (uniform) case 3, for a fixed number of taps it has a longer impulse response and thus better-defined spectral features. The nonuniform filter gains this advantage by trading total controllable bandwidth for greater precision over the smaller region.

So when is nonuniform synthesis practical? Without complicated clocking it seems inevitable that a nonuniform system will be

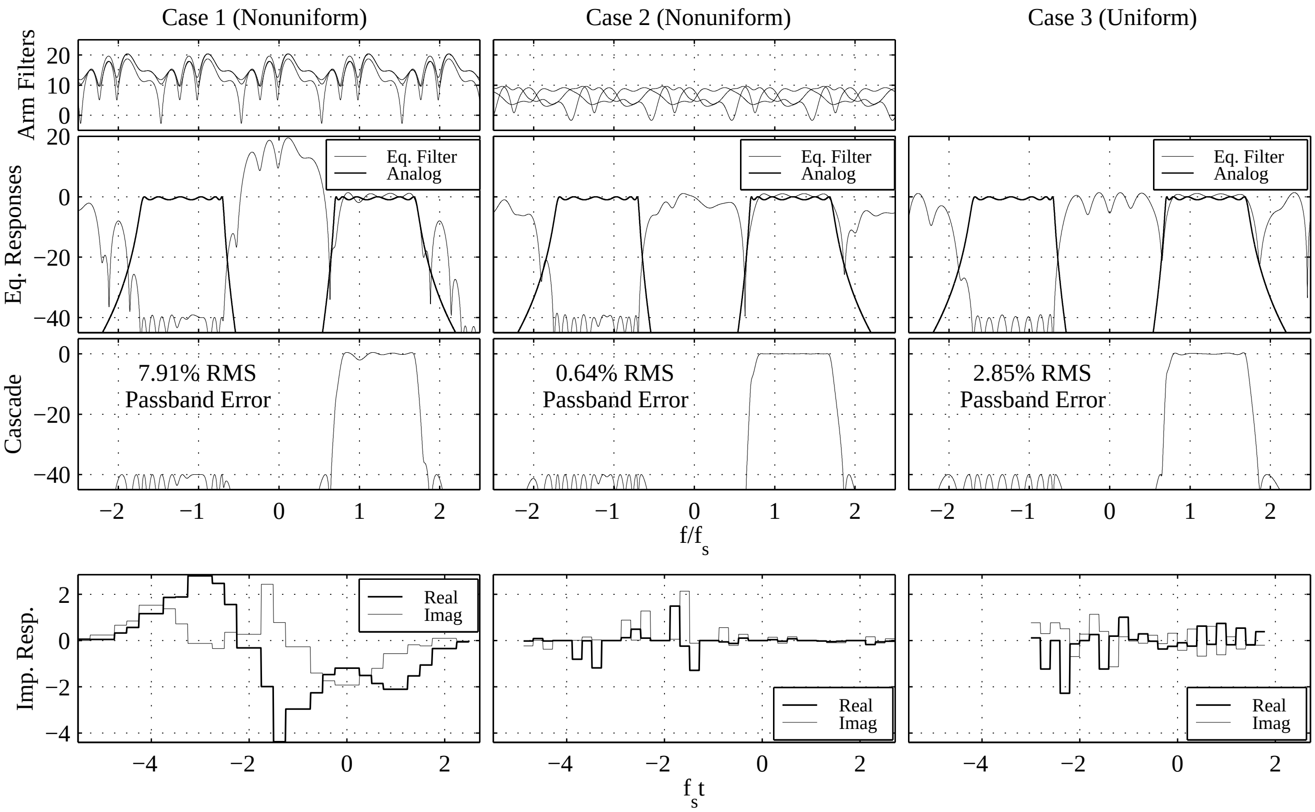


Figure 6: Arm-, analog-, and equivalent-filter and cascade magnitude responses and equivalent-filter impulse responses for the example.

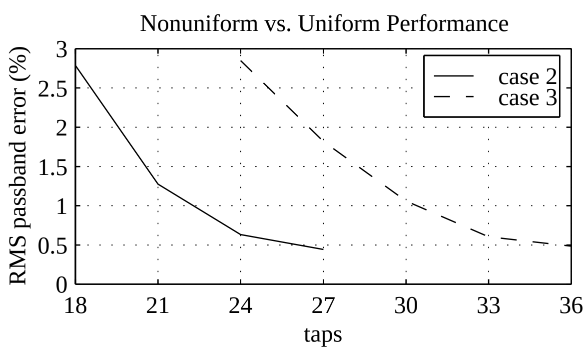


Figure 7: RMS passband error vs. filter length for cases 2 and 3.

realized as a thinned version of some higher-rate uniform system. A nonuniform system is generally no less demanding of D/A speed and clocking than a higher-rate uniform system. Given ample DSP power, the uniform system is the simple choice. If, however, the driving requirement is to minimize DSP computation, then nonuniform synthesis may offer better performance.

5. REFERENCES

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