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OPTIMAL FILTERS FOR RANGE-TIME SIDELOBE SUPPRESSION

Dan P. Scholnik

scholnik@nrl.navy.mil

Naval Research Laboratory
Radar Division
Washington, DC

ABSTRACT

Powerful convex optimization techniques can be applied to the design of the system pulse in radar systems. The flexibility of direct optimization allows control over such parameters as the beamwidth, maximum sidelobe level, and total sidelobe energy. Instead of designing the entire receive filter, a short filter is optimized in cascade with a matched filter, allowing the use of pulse-compression waveforms of arbitrary length. The extension to multiple-pulse synthetic-wideband waveforms is also presented.

1. INTRODUCTION

Frequently in pulsed radar systems, especially pulse-compression systems, we want to control the range-time sidelobe characteristic associated with a given waveform. For certain classes of waveforms, notably linear FM (chirp) signals, this is typically accomplished by the choice of a frequency-response weighting function applied to the transmitted signal, its matched filter, or both. The drawback of such an approach is that the usual weighting functions are not optimal in any sense, or offer limited flexibility to customize the system pulse. Further, inevitable analog system responses will corrupt the pulse, and while the system may be equalized separately, independent equalizer design divorced from actual system constraints usually results in overdesign. It is desirable to use optimization techniques developed for general filter design [1, 2, 3] and communication receiver design [4, 5] to optimize the system pulse directly, incorporating analog system responses.

In the most-common case where all sidelobe suppression is performed in the receiver, we can use convex optimization (in this case, second-order cone programming [6, 7]) to design an FIR filter that controls the system pulse characteristics for a given waveform. The straightforward approach of designing a single receive filter is often impractical, as typical pulse-compression waveform durations can far exceed the capability of even ultra-efficient algorithms. Therefore, we assume that a matched filter already exists in the system, and the filter under design operates not on the received waveform but on the compressed output of the matched filter. The result is that the required length of the filter is largely independent of the waveform length, and depends mainly on the degree of amplitude shaping needed and the amount of amplitude and phase distortion introduced by the system.

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2. SYSTEM ANALYSIS

Figure 1 shows a simple model of a pulse radar system. Conceptually, an impulse is fed to a filter with impulse response $w(t)$, creating the baseband waveform. This waveform is upconverted in frequency and transmitted. After reflecting off a target (here assumed stationary for the pulse duration) the received waveform is converted back down to baseband and matched-filtered. The matched filter output is then sampled (the sampling may in fact occur closer to the antenna) at a rate $f_s = 1/T_s$ and filtered by the sidelobe suppression filter with impulse response $h[n]$. Sampling offset τ represents the unknown (fractional) delay corresponding to the target location. The finite bandwidth of the system results in an output pulse of nonzero width.

This system can be simplified considerably by using a baseband-equivalent model and combining responses. Convolving $w(t)$ and $w^*(-t)$ results in the autocorrelation function $R_w(t)$. All other system responses (primarily analog filters) can be modeled by a single baseband-equivalent filter with complex impulse response $g(t)$. The result is seen in the top system of Fig. 2. Defining tapped-delay-line response $h(t) = \sum_n h[k]\delta(t - nT_s)$ allows the exchange of the sampler and digital filter to obtain the bottom system, which clearly shows the system pulse as

$$\begin{aligned} p(t) &= (R_w \circledast g \circledast h)(t) \\ &= \textstyle\sum_n h[n]\big(R_w \circledast g\big)(t - nT_s), \end{aligned}$$

the convolution of known responses with the filter to be designed. This is linear in the coefficients $h[k]$, ideal for convex program formulations.

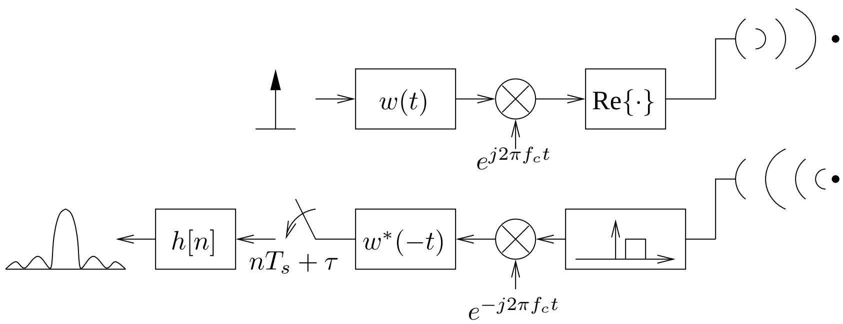

Figure 1: A simple model of a pulsed radar system.

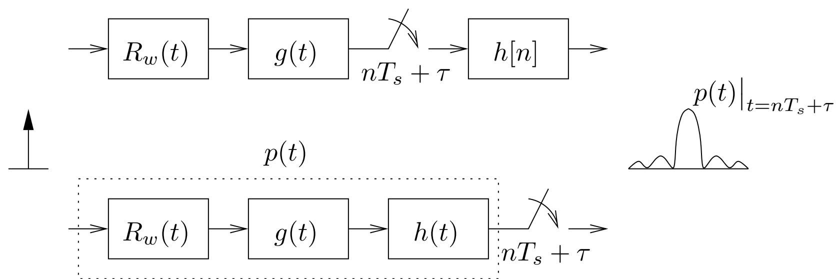

Figure 2: Two equivalent baseband systems are shown above. Combining waveform and analog system responses (top) and exchanging the sampler with the digital filter (bottom) make the system pulse readily apparent.

3. SINGLE-PULSE EXAMPLE

For this simple example we assume a linear FM waveform with bandwidth $B = 0.8 f_s$ and a high time-bandwidth product, so that we can well approximate [8] its autocorrelation as $R_w(t) \approx \mathrm{sinc}(Bt)$ and its power spectrum as $S_w(f) \approx 1/B\,\mathrm{rect}(f/B)$. Two identical bandpass filter responses derived from fifth-order Chebychev prototypes are used as a typical analog response. The resulting system pulse (top of Fig. 3) before correction has high sidelobes and is asymmetric due to the analog filtering. The filter to be designed is a length-11, complex-coefficient, nonlinear-phase FIR.

A straightforward approach to optimizing $p(t)$ is to fix the gain in the center, and minimize the maximum sidelobe level outside of some interval about the center:

$$\begin{aligned} \text{min.} \quad & \delta \\ \text{s.t.} \quad & |p(t)| \le \delta,\ t \in \mathcal{T}_{\mathrm{sl}} \\ & \mathrm{p}(0) = 1 \end{aligned} \tag{1}$$

The middle plot of Fig. 3 shows the optimized system pulse with $\mathcal{T}_{\mathrm{sl}}$ a uniform grid of times over the region $[-50T_s, 50T_s] \setminus (-2.4T_s, 2.4T_s)$ with a grid spacing of $T_s/10$. FFT-based techniques were used to efficiently calculate system pulse values on a uniform grid with spacing T_s/M, with M an integer. Using a higher constraint grid density than the sampling density allows greater precision in choosing the mainlobe width and ensures that the resulting pulse is insensitive to the sampling offset τ. (Too coarse of a grid allows the optimization to place nulls near each grid point, for example.) In the optimization, each grid point represents a single second-order cone constraint of rank 2. The optimal pulse has a uniform sidelobe level of -44.8 dB about the center, eventually rolling off slowly. To improve both the sidelobe rolloff rate and the total sidelobe energy, we modify the previous optimization:

$$\begin{aligned} \min \quad & \delta \\ \text{s.t.} \quad & |p(t)| \le 10^{-40/20},\ t \in \mathcal{T}_{\mathrm{sl}} \\ & \int_{\mathrm{t} \in \mathcal{T}_{\mathrm{sl}}} |p(t)|^2\, dt \le \delta \\ & p(0) = 1 \end{aligned} \tag{2}$$

Now the total sidelobe energy is minimized, subject to a -40 dB maximum sidelobe level. The same grid of frequencies was used for $\mathcal{T}_{\mathrm{sl}}$, and the integral shown was approximated by a Riemann sum over the grid, adding a single high-rank second order cone constraint to the previous problem. The resulting system pulse is shown in the bottom plot of Fig. 3. The mainlobe response is essentially unchanged, but now the sidelobes roll off faster, and are in fact lower over most of the sidelobe region. By giving up a small amount of sidelobe suppression close to the mainlobe, we gain a large savings farther out, an argument against pure equiripple designs that has parallels in array pattern design and more traditional filter design [1]. The system frequency responses for this case are shown in Fig. 4. Even without any frequency-domain constraints, the group delay of the system pulse is essentially flat.

That the "equiripple" design has only a few sidelobes of equal height is due to the short sidelobe suppression filter, since the analytical solution to the Chebychev (equiripple) problem in this case has sharp frequency response features

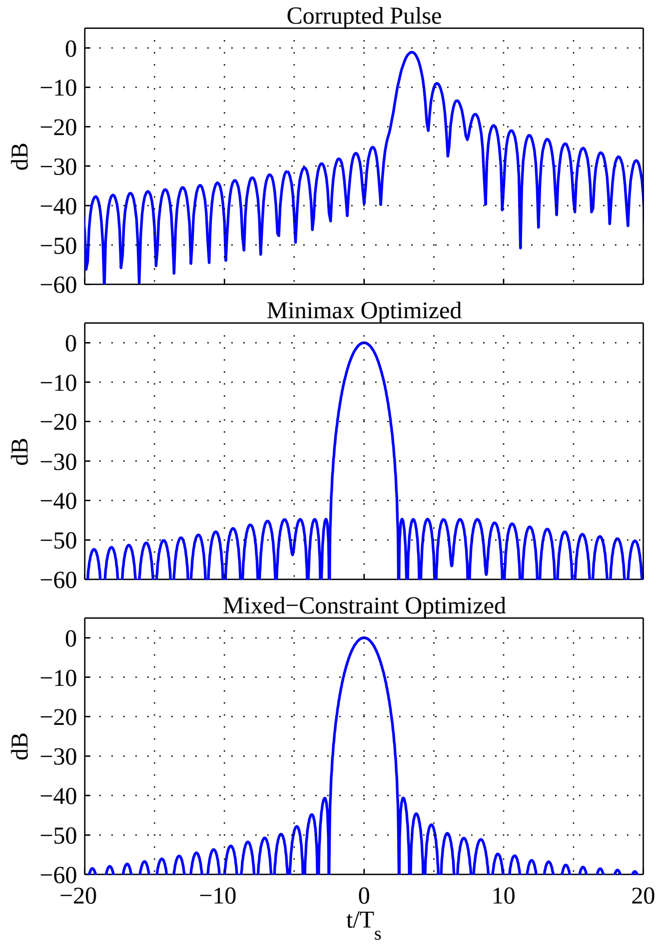

Figure 3: Corrupted and optimized system pulses using minimax and mixed minimax/energy constraints.

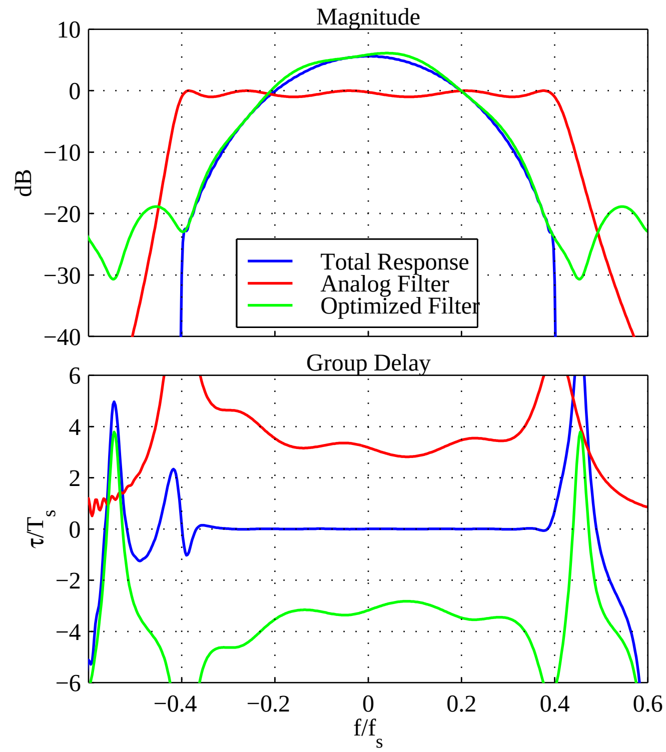

Figure 4: System frequency responses for the mixed-constraint example.

and thus a long impulse response. Increasing the filter length produces a corresponding increase in the number of equal-height sidelobes but barely improves the sidelobe level at all, with a lower limit around -46 dB, while actually increasing total sidelobe energy. This also has a parallel in array pattern design, where Chebychev weightings are generally shunned in favor of Taylor and similar weightings in the spirit of the mixed-constraint example above.

4. SYNTHETIC WIDEBAND EXAMPLE

An interesting extension of the single pulse case is synthetic wideband, where several narrow-bandwidth pulses transmitted at different RF center frequencies are combined to form a single high-bandwidth pulse. Figure 5 (top) shows such a system, where $p_1(t), \ldots, p_K(t)$ represent baseband-equivalent system pulses as before corresponding to RF center frequencies $f_c + f_1, \ldots, f_c + f_K$, and the sampling rate has been increased by an integer factor M to accommodate the higher total bandwidth. We can again rearrange the system (Fig. 5 bottom) so that the composite system pulse is seen to be

$$p(t) = \sum_k p_k(t) e^{j2\pi f_k t},$$

where (assuming identical baseband waveforms and analog filtering)

$$p_k(t) = \sum_n h_k[n] \big(R_w \circledast g \big)(t - nT_s).$$

Note that the nonlinear-phase, complex-coefficient sidelobe suppression filters $h_1, \ldots, h_K$ retain the T_s coefficient spacing of the single-pulse case, since the individual pulse bandwidths are unchanged. Since $p(t)$ is also linear in the coefficients of the component filters, the same procedure as before can be used to optimize the overall system pulse.

As an example, consider a five-pulse synthetic waveform using the same bandwidth $B = 0.8 f_s$ linear FM pulse and Chebychev analog filter as before and stepping by $0.8B = 0.64 f_s$ between pulses for a total bandwidth of $4.2B = 3.36 f_s$. The choice of M has no effect on the design, but must be greater than 4 to avoid aliasing. The top plot of Fig. 6 shows the system pulse optimized according to (2), with the mainlobe cutout region defined as $(-0.55T_s, 0.55T_s)$. Experimentation yielded good results with filter lengths of 17, with shorter lengths resulting in artifacts in the far sidelobes.

In practice the sampling rate for each return would be chosen based on the individual pulse bandwidths to minimize computation in the digital filtering, and after filtering the pulses would be interpolated to a common higher sampling rate. Assuming bandlimited waveforms and a "harmless" interpolator, the two are equivalent.

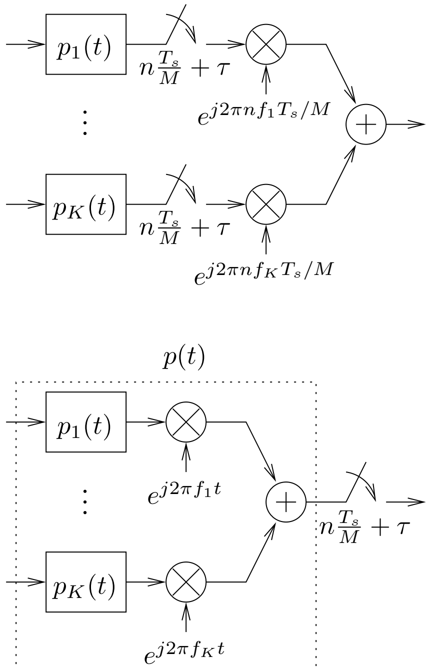

Figure 5: A simplified synthetic-wideband system (top) and an equivalent rearrangement (bottom) showing the composite system pulse $p(t)$.

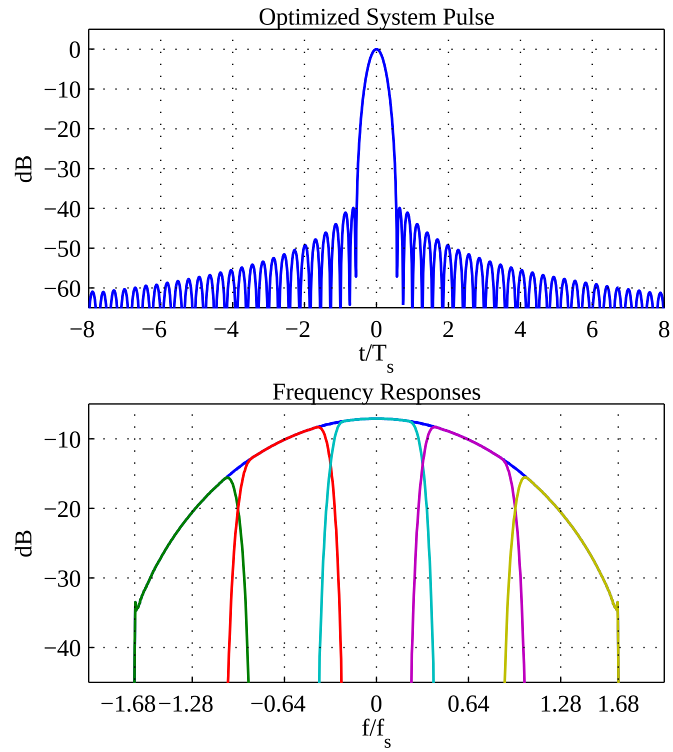

Figure 6: System pulse and frequency response plots for the synthetic wideband example.

The bottom plot of Fig. 6 shows the frequency responses of the five filtered waveforms and their sum, which has the smooth, tapered shape of a single pulse as expected.

5. CONCLUSIONS

This paper presents an optimization-based approach to controlling the sidelobe structure of pulse-compressed returns in both a typical single-waveform radar and a synthetic-wideband multiple-waveform radar. FIR filters of modest length can be designed to optimize the system pulse while equalizing analog system responses. Today's efficient interior point engines encourage experimentation and custom design—the SeDuMi engine required just seven seconds to optimize the first single pulse design on a 450MHz PC, for example, compared to thirteen seconds for the problem setup.

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